



奧冠教育中心

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香港國際數學競賽初賽 2019 (香港賽區)

HONG KONG INTERNATIONAL
MATHEMATICAL OLYMPIAD

HEAT ROUND 2019 (HONG KONG REGION)

中學二年級 Secondary 2

時限: 60 分鐘

Time allowed: 60 minutes

試題

Question Paper

考生須知:

Instructions to Contestants:

1. 本卷包括 試題 乙份, 試題紙不可取走。
Each contestant should have ONE Question-Answer Book which CANNOT be taken away.
2. 本卷共 5 個範疇, 每範疇有 4 題, 共 20 題, 每題 3 分, 總分 60 分, 答錯不扣分。
There are 5 exam areas and 4 questions in each exam area. There are a total of 20 questions in this Question-Answer Book. Each carries 3 marks. Total score is 60 marks. No points are deducted for incorrect answers.
3. 請將答案寫在 **答題紙** 上。
All answers should be written on ANSWER SHEET.
4. 比賽期間, 不得使用計算工具。
NO calculators can be used during the contest.
5. 本卷中所有圖形不一定依比例繪成。
All figures in the paper are not necessarily drawn to scale.
6. 比賽完畢時, 本試題會被收回。
This Question-Answer Book will be collected at the end of the contest.

請以最簡形式填寫答案, 若計算結果是分數, 請確保為真分數或帶分數, 或將計算結果寫成小數。錯誤單位將不給予任何分數。
Write down the answer in the simplest form. If the calculation result is a fraction, please write down the answer as a proper or mixed fraction, decimal figure is also accepted. Marks will NOT be given for incorrect unit.

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本試題不可取走。

THIS Question-Answer Book CANNOT BE TAKEN AWAY.

未得監考官同意，切勿翻閱試題，否則參賽者將有可能被取消資格。

DO NOT turn over this Question-Answer Book without approval of the examiner.

Otherwise, contestant may be DISQUALIFIED.

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填空题 (第 1 至 20 題) (每題 3 分, 答錯及空題不扣分)

Open-Ended Questions (1st ~20th) (3 points for correct answer, no penalty point for wrong answer)

Logical Thinking

邏輯思維

1. Given A and B are two non-zero digits and the 3-digit numbers formed by these two digits have the following properties:

1. \overline{BAB} is divisible by 11;

2. \overline{BBA} is divisible by 7;

Find the maximum value of 3-digit number \overline{ABB} .

已知 A 和 B 為兩個非零數位、且利用這兩個數位組成的三位數有以下性質:

1. \overline{BAB} 可以被 11 整除;

2. \overline{BBA} 可以被 7 整除;

求三位數 \overline{ABB} 的最大值。

2. Alice goes northeast for 80km, then goes southeast for 48km and goes southwest for 60km. How far is she now from the original position?

愛麗絲向東北走 80 公里, 向東南走了 48 公里, 向西南走了 60 公里, 問她和原來位置相距多遠?

3. If $\overline{abcd} + \overline{badc} = 12254$, calculate $a + b + c + d$.

若 $\overline{abcd} + \overline{badc} = 12254$, 求 $a + b + c + d$ 的值。

4. Now is 12 o'clock. How long does it take to let the minute hand and hour hand of the clock to form a right-angle?

現在是十二時, 多久之後時針和分針才會形成直角?

Algebra

代數

5. Find the value of y if $|3x - y + 1| + |x + y + 1| = 0$.

若 $|3x - y + 1| + |x + y + 1| = 0$, 求 y 的值。

6. Factorize $x^4 + 9y^4 + 2x^2y^2$.

因式分解 $x^4 + 9y^4 + 2x^2y^2$

7. How many negative integral solution(s) is / are there for x if $-98 \leq 13x + 6 \leq 71$?

若 $-98 \leq 13x + 6 \leq 71$, 則 x 有多少個負整數解?

8. Given that $a > 5$ and the equation $ax^2 + 8x - 3c = 0$ has real root(s). What is the minimum value of

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integer c ?

已知 $a > 5$ 且方程 $ax^2 + 8x - 3c = 0$ 有實根。整數 c 最小是多少?

Number Theory

數論

9. Find the remainder when 520^{205} is divided by 7.
求當 520^{205} 除以 7 時所得的餘數。
10. x is a positive integer such that the remainder of x^2 divided by 13 is 9. If $x > 31$, what is the minimum value of x ?
 x 是一個正整數使 x^2 除以 13 餘 9。若 $x > 31$, x 最小是多少?
11. Find the last digit of C if $C = 1 + 3 + 3^2 + 3^3 + \dots + 3^{887} + 3^{888}$
若 $C = 1 + 3 + 3^2 + 3^3 + \dots + 3^{887} + 3^{888}$, 求 C 的個位值。
12. Find the number of positive integer(s) smaller than 1331 that is / are co-prime with 1331.
求小於 1331 而與 1331 互質的正整數數目。

Geometry

幾何

13. Given that right-angled $\triangle ABC \sim \triangle DEF$ and two right-angled sides $AB = 8$, $AC = 15$. Find $\frac{DE}{EF}$.
已知直角三角形 $\triangle ABC \sim \triangle DEF$ 且兩直角邊 $AB = 8$, $AC = 15$ 。求 $\frac{DE}{EF}$ 。
14. For three points on a coordinate plane $A(-1, 3), B(0, -3), C(8, 5)$, find the area of the triangle formed by using those three points as vertices.
直角坐標上有三個點分別為 $A(-1, 3), B(0, -3), C(8, 5)$, 求以該三點為頂點的三角形的面積。
15. A cone is formed by a circles and a semi-circle and the surface area is 4158cm^2 . Find the radius of the semi-circle. (Take $\pi = \frac{22}{7}$)
一個圓柱錐體由一個圓形和一個半圓組成, 表面面積為 4158 立平方厘米, 求半圓的半徑。(取 $\pi = \frac{22}{7}$)
16. An exterior angle of a n -sided polygon is $(3n - 21)^\circ$. Find n .
一個正 n 邊形的其中一隻外角為 $(3n - 21)^\circ$, 求 n 。

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Combinatorics

組合數學

17. In how many possible ways can 12 identical flowers be distributed to 4 distinct vases with at least one in each vase?
把 12 支完全相同的花放入四個不同的花瓶中，每個花瓶至少有 1 支花，問共有多少個不同的分配方法？
18. There are 6 identical blue boxes and 3 identical red boxes are put from left to right. How many way(s) of arrangement is / are there?
由左至右放置 6 個相同的藍色箱子和 3 個相同的紅色箱子，有多少個不同排列方法？
19. A fair 6-face die is thrown 3 times. Find the probability that the sum of numbers obtained is greater than 6.
擲一枚均質六面骰子三次。求擲得點數之和大於 6 的概率。
20. For a n -digit number which leftmost digit is 5, if the leftmost digit is now put at the rightmost, the new number formed is 3 times the original. Find the minimum value of n .
有一個 n 位數的最左邊數位是 5，若把最左邊的數位放在最右邊，則組成的數字會是原來的 3 倍。求 n 的最小值。

~ 全卷完 ~

~ End of Paper ~

請以最簡形式填寫答案，若計算結果是分數，請確保為真分數或帶分數，或將計算結果寫成小數。錯誤單位將不給予任何分數。

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